Solution 1 (40 points)

1. **First calculate the diffusion coefficient:**

$$D_{12} = \frac{1.86 \times 10^{-3} \times T^{1.5} \times (\frac{1}{M_1} + \frac{1}{M_2})^{0.5}}{P{\sigma_{12}}^2\Omega}$$

Check the table we have:

$$\sigma_{Ar} = 3.542, \sigma_{He} = 2.551$$

$$\frac{\varepsilon_{Ar}}{k_B} = 93.3, \frac{\varepsilon_{He}}{k_B} = 10.2$$

We have:

$$\sigma_{12} = \frac{\sigma_{Ar} + \sigma_{He}}{2} = 3.046$$

$$\frac{\varepsilon_{12}}{k_B} = \sqrt{\frac{\varepsilon_{Ar}}{k_B}\frac{\varepsilon_{He}}{k_B}} = 30.85$$

$$\frac{Tk_B}{\varepsilon_{12}} = 3.24 \ \rightarrow \ \Omega = 0.92$$

Calculate the diffusion coefficient:

$$D_{12} = \frac{1.86 \times 10^{-3} \times 100^{1.5} \times (\frac{1}{4} + \frac{1}{40})^{0.5}}{5 \times 3.046^2 \times 0.92} = 0.0228\ cm^2/s$$

2. **Convection is neglected:**

$$\frac{C(z,t) - C_s}{C_\infty - C_s} = \operatorname{erf}(\xi), \xi = \frac{z}{\sqrt{4Dt}} = \frac{0.009}{\sqrt{4 \times 0.00000228 \times 36}} = 0.497$$

$$C(z,t) = C_s \times (1 - \operatorname{erf}(\xi)) = C_s(1 - 0.52) = 0.48C_s$$

$$P = 0.48P_s = 0.53\ bar \times 0.48 = 0.254\ bar$$

3. **Including convection:**

$$\bar{V}_1 {c_1}^{sat} = (1 + \frac{1}{\sqrt{\pi}(1 + erf\phi)\phi e^{\phi^2}})^{-1}$$

$$\bar{V}_1 {c_1}^{sat} = \frac{{c_1}^{sat}}{c_{total}} = {y_1}^{sat} = \frac{0.53}{5} = 0.106$$

$$0.106 = (1 + \frac{1}{\sqrt{\pi}(1 + erf\phi)\phi e^{\phi^2}})^{-1}$$

$$(1 + erf\phi)\phi e^{\phi^2} = 0.0668$$

Solve for ϕ: $\phi = 0.063$

$$\frac{c_1}{{c_1}^{sat}} = \frac{1 - \operatorname{erf}(\xi - \phi)}{1 + \operatorname{erf}(\phi)} = \frac{1 - \operatorname{erf}(0.497 - 0.063)}{1 + \operatorname{erf}(0.063)} = \frac{1 - 0.46}{1 + 0.07} = 0.505$$

$$c_1 = 0.505{c_1}^{sat}$$

$$p_1 = 0.505P_s = 0.505 \times 0.53\ bar = 0.267\ bar$$

The error is:

$$error = \frac{0.254 - 0.267}{0.267} = -4.9\%$$

Solution 2 (30 points)

$$\frac{dn_1}{dz} = 0$$
$$\frac{dn_2}{dz} = 0$$

n_1 and n_2 are constant.

Let's look at the boundary $z = 0$, $n_2 = -3n_1$

$$n = n_1 + n_2 = -2n_1 = cv$$
$$c = c_1^0 + c_2^0 = c_1^L + c_2^L$$
$$c_1 = y_1 c$$
$$n_1 = -Dc\nabla y_1 + c_1 v$$

Substitute variables

$$n_1 = -Dc\nabla y_1 + y_1 n = -Dc\nabla y_1 - 2y_1 n_1$$
$$n_1(1 + 2y_1) = -Dc\frac{dy}{dz}$$

Transpose and integrate

$$\int_0^L \frac{n_1}{Dc} dz = -\int_{y_1^0}^{y_1^l} \frac{dy}{(1 + 2y_1)}$$

$$\frac{n_1 L}{Dc} = -\frac{1}{2}\ln(\frac{1 + 2y_1^L}{1 + 2y_1^0})$$

$$n_1 = \frac{Dc}{2L}\ln(\frac{1 + 2y_1^0}{1 + 2y_1^L})$$

$$n_2 = -\frac{3Dc}{2L}\ln(\frac{1 + 2y_1^0}{1 + 2y_1^L})$$

Since

$$v_2^L = \frac{n_2^L}{c_2^L}$$

Plug in the variables

$$v_2^L = -\frac{3D(c_1^L + c_2^L)}{2Lc_2^L}\ln\left(\frac{1 + 2y_1^0}{1 + 2y_1^L}\right) = -\frac{3D}{2L}(1 + \frac{c_1^L}{c_2^L})\ln\left(\frac{c + 2c_1^0}{c + 2c_1^L}\right)$$

Or

$$n_1 = -\frac{1}{3}n_2$$

$$n = n_1 + n_2 = \frac{2}{3}n_2 = cv$$

$$n_2 = -Dc\nabla y_2 + y_2 n = -Dc\nabla y_2 + \frac{2}{3}y_2 n_2$$

$$n_2(1 - \frac{2}{3}y_2) = -Dc\nabla y_2$$

$$\int_0^L \frac{n_2}{Dc} dz = -\int_{y_2^0}^{y_2^l} \frac{dy}{(1-\frac{2}{3}y_2)}$$

$$n_2 = \frac{3Dc}{2L} \ln(\frac{1-\frac{2}{3}y_2^L}{1-\frac{2}{3}y_2^0})$$

$$v_2^L = \frac{n_2^L}{c_2^L} = \frac{3Dc}{2Lc_2^L} \ln(\frac{1-\frac{2}{3}y_2^L}{1-\frac{2}{3}y_2^0}) = \frac{3D}{2L}(1+\frac{c_1^L}{c_2^L}) \ln(\frac{3c-2c_2^L}{3c-2c_2^0})$$

Solution 3 (30 points)

Build the mass balance on the bubble:

$$\frac{d}{dt}(c_1 V) = -K_p A(P_1 - H \times c_{1,liq})$$

c_1 is the concentration of gas in the bubble, $c_{1,liq}$ is the concentration of bulk solution. K_p is the gas side total mass transfer coefficient, A is the surface area of the bubble, H is the Henry constant.

Since the reservoir is well-mixed, $c_{1,liq} = 0$

$$\frac{d}{dt}(c_1 V) = -K_p A(P_1 - 0)$$

$$\frac{d}{dt}(c_1 V) = \frac{dr}{dt}(c_1 \times 4\pi r^2) = -K_p \times 4\pi r^2 \times P_1$$

$$\frac{dr}{dt} = -K_p P_1 / c_1$$

Integration we have:

$$r - r_0 = -\frac{K_p P_1}{c_1} \times (t - t_0)$$

Boundary conditions:

$$t_0 = 0\ s, r = r_0$$

$$t_0 = 1\ s, r = (1 - 10\%) r_0$$

We have:

$$(1 - 10\%) r_0 - r_0 = -\frac{K_p P_1}{c_1} \times 1\ s$$

$$-10\% r_0 = -\frac{K_p P_1}{c_1} \times 1\ s$$

$$K_p = \frac{10\% r_0 \times c_1}{P_1 \times 1s}$$

$$\frac{c_1}{P_1} = \frac{1}{RT}$$

$$K_p RT = \frac{10\% r_0}{1s} = 0.0005\ m/s$$

$$K_p = \frac{10\% r_0}{RT \times 1s} = \frac{0.1 \times 0.005\ m}{8.314 \frac{J}{mol \cdot K} \times 298K \times 1s} = 2 \times 10^{-7}\ \frac{mol}{Pa \cdot m^2 \cdot s}$$